

Precalc Warm Up # 10-3

1. Find $\sum_{k=3}^{208} (5k + 4)$

2. 30 numbers are in an Arithmetic Sequence. The sum of the numbers is 1020 and the first number is 5. Give the last number.

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EXERCISES 8.1.3 - MISCELLANEOUS QUESTIONS

1. Find the twentieth term in the sequence 9, 15, 21, 27, 33, ...

2. Fill the gaps in this arithmetic sequence: $-3, _, _, _, _, _, 12$.

3. An arithmetic sequence has a tenth term of 17 and a fourteenth term of 30. Find the common difference.

$$a_{10} = 17$$

$$a_{14} = 30$$

n	10	14
a_n	17	30

use $d = \text{slope} \rightarrow d = \frac{30-17}{14-10}$
 $d = \frac{13}{4}$

or from $n = 10$ to 14 , you have
 4 jumps of d :

$$17 + 4d = 30$$

↓

4. If $u_{59} = \frac{1}{10}$ and $u_{100} = -1\frac{19}{20}$ for an arithmetic sequence, find the first term and the common difference.

$$\begin{array}{c|ccc} n & \dots & 59 & \dots & 100 \\ \hline a_n & & 0.1 & & -1.95 \end{array}$$

$(59, 0.1) \quad (100, -1.95)$

$$d = \frac{-1.95 - 0.1}{100 - 59}$$

$$d = -0.05$$

$$d = -\frac{1}{20}$$

$$a_n = a_1 + d(n-1)$$

$$\frac{1}{10} = a_1 - \frac{1}{20}(58)$$

↓

$$a_1 = 3$$

5. Find the sum of the first one hundred odd numbers.

6. An arithmetic series has twenty terms. The first term is -50 and the last term is 83 , find the sum of the series.

7. Thirty numbers are in arithmetic sequence. The sum of the numbers is 270 and the last number is 38 . What is the first number?

8. How many terms of the arithmetic sequence: 2, 2.3, 2.6, 2.9, ... must be taken before the terms exceed 100?

n

$$a_1 + d(n-1) = a_n$$

$$2 + 0.3(n-1) > 100$$

$$n > 327.\bar{6}$$

\therefore 328 terms must be taken before the term exceeds 100.

9. Brian and Melissa save \$50 in the first week of a savings program, \$55 in the second week, \$60 in the third and so on, in arithmetic progression. How much will they save in ten weeks? How long will they have to continue saving if their target is to save \$5000?

Week	1	2	3
n	1	2	3
\$	50	55	60

$$5000 = \frac{n}{2} [2(50) + 5(n-1)]$$

$$10000 = n(100 + 5n - 5)$$

$$S_{10} = \frac{10}{2} [2(50) + 5(9)]$$

\downarrow

$$0 = 5n^2 + 95n - 10,000$$

$$0 = n^2 + 19n - 2000$$

$$n = \frac{-19 \pm \sqrt{19^2 - 4(-2000)}}{2}$$

$$361 + 8000$$

$$n = \frac{-19 \pm \sqrt{8361}}{2}$$

$$n \approx 36.2$$

They need to save for 37 weeks

12. Find each sum, ** Number 11 follows the assignment slide.*

(a) $\sum_{k=1}^{100} k$

(b) $\sum_{k=1}^{100} (2k+1)$

(c) $\sum_{k=1}^{51} (3k+5)$

↑
Gauss's problem
!!

13. If $u_i = -3 + 4i$ and $v_i = 12 - 3i$ find

(b) $\sum_{i=1}^{10} (3u_i + 4v_i)$

$$\sum_{i=1}^{10} [3(-3+4i) + 4(12-3i)]$$

$$\sum_{i=1}^{10} [-9+12i+48-12i]$$

$$\sum_{i=1}^{10} 39 = 39(10) = \boxed{390}$$

Groups, Brief Review:

1. $44286.75 = 0.25 (3)^{n-1}$

2. $\sum_{k=25}^{100} 5$

3. The fifth term of a geometric sequence is 3 and the seventh term is 0.75. Find the common ratio and the first term.

Groups, Brief Review:

1. $\frac{44286.75}{0.25} = \frac{0.25 (3)^{n-1}}{0.25}$

$177147 = 3^{(n-1)}$

$\ln 177147 = \ln 3^{(n-1)}$

$\frac{\ln 177147}{\ln 3} = \frac{(n-1)(\ln 3)}{\ln 3}$

$11 = n-1$

$n = 12$

2. $\sum_{k=25}^{100} 5$

$n = 100 - 25 + 1$
 $= 76$

$\sum_{k=25}^{100} 5 = 5(76)$
 $= 380$

3. The fifth term of a geometric sequence is 3 and the seventh term is 0.75. Find the common ratio and the first term.

$a_5 = 3 \quad a_7 = 0.75$

n	5	7
	3	0.75

$t(n) = a(r^n)$

↑
zero term

3. The fifth term of a geometric sequence is 3 and the seventh term is 0.75. Find the common ratio and the first term.

Intuitive approach:

$$3r^2 = 0.75$$

$$r^2 = 0.25$$

$$r = \pm 0.5$$

n	5	7
g_n	3	0.75

2 jumps of r

$$g_1(r)^4 = 3$$

$$g_1(\pm 0.5)^4 = 3$$

$$g_1(0.0625) = 3$$

$$g_1 = 48$$

n	1	2	3	4	5
g_n	g_1				3

4 jumps of r

GEOMETRIC SEQUENCE, GP

has a CONSTANT RATIO, r , instead of a constant difference

n	1	2	3	4
g_n	2	6	18	54

find 15th term

$r = 3$

$$g_2 = g_1 r$$

$$g_2 = 2(3)^1 = 6$$

$$g_3 = 2(3)^2 = 18$$

$$g_4 = 2(3)^3 = 54$$

So the power on r is $(n-1)$

$$g_n = g_1 r^{(n-1)}$$

$$g_{15} = 2(3)^{14}$$

$$g_{15} = 9,565,938$$

A GP has 5th term 3 and 7th term 0.75. Find the first term, the common ratio, and the tenth term.

from earlier...

$$g_1 = 48$$

$$r = \pm 0.5$$

$$g_n = g_1(r)^{n-1}$$

$$g_{10} = 48(\pm 0.5)^9$$

$$g_{10} = \pm \frac{3}{32}$$

★ So there are 2 sequences described here. The terms where n is even could either be negative or positive.

How many terms are there in the GP below?

0.25, 0.75, 2.25, ..., 44286.75

$$r = \frac{0.75}{0.25}$$

$$r = 3$$

$$g_n = g_1(r)^{n-1}$$

$$44286.75 = 0.25(3)^{n-1}$$

we solved this earlier!

$$\boxed{n=12}$$

Find x if these are three consecutive terms in a GP.

$$6, x, 96 \quad \frac{x}{6} = \frac{96}{x}$$

$$\frac{x}{6} = \frac{96}{x}$$

$$\sqrt{x^2} = \sqrt{576}$$

$$x = \pm 24$$

Find k if these are three consecutive terms in a GP:

$$k + 2, 3k + 1, 7k - 1$$

Use common ratio!

★ 2 different sequences are possible 😊

$$\frac{3k+1}{k+2} = \frac{7k-1}{3k+1}$$

Now cross multiply.

$$\begin{aligned} &\vdots \\ &\downarrow \\ &2k^2 - 7k + 3 = 0 \\ &(2k-1)(k-3) = 0 \end{aligned}$$

$$k = \frac{1}{2}, 3$$

A person on a diet want to lose 1% of their body weight each week. If they start at 200 pounds, write a geo seq that gives the week by week weight of the person. Find their weight after 6 weeks. When will they reach their goal weight of 170 pounds? find n

After 1 wk After 2 wks $r \Rightarrow 100\% - 1\% = 99\%$
 198 196.02 so: $r = 0.99$

\uparrow
 g_1 $g_6 = 198(0.99)^5$
 $g_6 \approx 188.3 \text{ lbs.}$

Goal of 170 lbs
 $198(0.99)^{n-1} = 170$

$(0.99)^{n-1} = \frac{170}{198}$ take \ln both sides.
 $\ln(0.99)^{(n-1)} = \ln\left(\frac{85}{99}\right) \leftarrow \left(\frac{170}{198} = \frac{85}{99}\right)$

$(n-1) \frac{\ln(0.99)}{\ln(0.99)} = \frac{\ln\left(\frac{85}{99}\right)}{\ln(0.99)}$

$n = \frac{\ln\left(\frac{85}{99}\right)}{\ln(0.99)} + 1$ Now calculate!

$n \approx 16.2$, so: 17 weeks

The number of people in a county increases by 2% per year. If the population at the start of 1970 was 12,300, what is the population at the start of the year 2011?

at start of 1970	at the start of 2011
term 1	term 42
12,300	g_{42}

$100\% + 2\%$
 102%
 $r = 1.02$

$g_{42} = 12,300(1.02)^{41}$
 $\approx 27,702$

2011
 - 1970
 + 1
 —————

HW: SL Book

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